

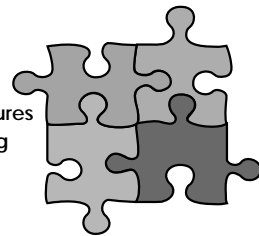
Rekenreks

Encouraging Math Talks

Gwen Hancock,
Program Specialist

DO THEY KNOW WHY?

"Our classrooms are filled with students and adults who think of mathematics as **rules and procedures to memorize without understanding the numerical relationships** that provide the foundation for these rules."



UNDERSTANDING

Math Talks

What is a Math Talk?

- ❖ A method to **elicit the thinking process** students use for problem-solving.
- ❖ Classroom conversations composed of purposefully crafted computation problems that are **solved mentally**.



Computational Fluency

Primary Goal of Math Talks

Computational Fluency = efficient and accurate methods for computing. When students demonstrate flexibility in the methods they choose, understand and explain these methods, and produce accurate answers efficiently.

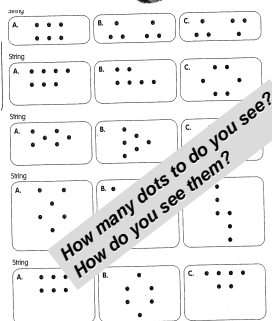
- ❖ Numbers are composed of smaller numbers
- ❖ Numbers can be taken apart and combined with other numbers to make new numbers
- ❖ What we know about one number can help us figure out other numbers
- ❖ Numbers are organized into groups of tens and ones, hundreds...
- ❖ What we know about numbers to 10 helps us with numbers to 100 and beyond

Number Sense

Math Talks

Develop Number Sense

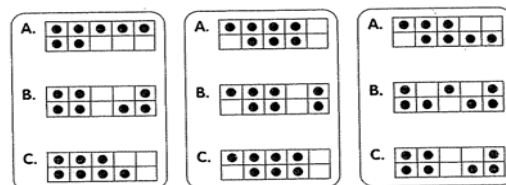
Number Sense = an understanding about what numbers are, their relationships, effect of operating on numbers, including mental math and estimation.



Fluency

Practice Fluency with Small Numbers

Fluency = knowing how a number can be composed and decomposed.



Subitize

Make use of Subitizing

Subitizing = To immediately recognize a group of objects as a single unit



- Comes from the Latin for "sudden"
- Recognizing the number of objects in a small collection requires no counting because the numerosity is identified in an instant.
- Works with four or fewer objects
- Subitization loses accuracy when the number of objects increases
- Helps children develop the abstract number and arithmetic strategies needed to master counting

Subitize

Conceptual and Perceptual Subitizing

1. **Perceptual Subitizing** -
 - Recognizing a number without using other mathematical processes
 - Helps children connect objects with one number word and develop the ability to count
2. **Conceptual Subitizing** -
 - Determine the number of a collection of objects by recognizing a familiar pattern
 - Spatial arrangement of dots on faces of dice or domino tiles
 - Using rhythmic patterns such as gesturing or one "beat" with each count

Addition Strategies

MULTIPLE STRATEGIES FOR ADDITION

- ❖ Counting All / Counting On
- ❖ Doubles / Near Doubles
- ❖ Making Tens
- ❖ Landmark or Friendly Numbers
- ❖ Compensation
- ❖ Break Each Number into its Place Value
- ❖ Adding Up in Chunks



6+7

What strategies do you use to solve?

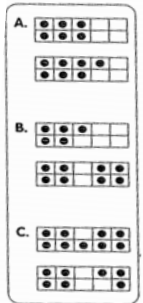
Not all tasks are created equal, and different tasks will provoke different levels of student thinking."

Stein, Smith, Henningsen & Silver, 2000

What different strategies will your students use to solve this problem?

$$\begin{array}{l} 3 + 3 \\ 3 + 4 \\ 3 + 2 \end{array}$$

$$\begin{array}{l} 4 + 4 \\ 4 + 3 \\ 4 + 5 \end{array}$$



Counting All/Counting On

Addition Strategies

Counting All -

- Kindergarten - 1st Grade
- Not able to add on, can not visualize a number and hold on to it in memory.

$$6 + 7 = \underline{\quad}$$

Child could count:: 1,2,3,4,5,6...

Or may choose to start with the number 6 and count on

Counting On -

- 1st - 2nd Grade
- Most efficient when counting up begins with largest number. $6 + 7 = \underline{\quad}$ 7, 8, 9, 10, 11, 12, 13



Doubles/Near Doubles

More Addition Strategies

Doubles/Near Doubles -

- Kindergarten ...when students are able to recall sums for many doubles
- Adjust one or both numbers to make a doubles or near-doubles combination

$$\begin{array}{l} 6 + 7 = 6 + (6 + 1) = \\ (6 + 6) + 1 = \\ 12 + 1 = 13 \end{array}$$

Do you see a property being used ...?

Compensation

How Many Ways to Add?

Compensation

- Strategy emerges in 1st grade, second semester
- Manipulate the numbers into friendly numbers
- Remove an amount from one addend and give that exact amount to the other addend.

$$\begin{array}{l} 19 + 6 \\ 9 + 16 \\ 9 + 26 \\ 29 + 6 \end{array}$$

$$\begin{array}{l} 7 + 19 \\ 5 + 29 \\ 39 + 8 \\ 49 + 6 \end{array}$$

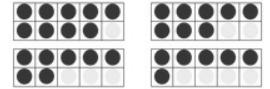
$$\begin{array}{l} 11 + 19 \\ 9 + 29 \\ 21 + 9 \\ 19 + 18 \end{array}$$

MAKING TENS

MAKING TENS

We want our students to be flexible with numbers. Making groups of tens develops understanding of our place value and number system.

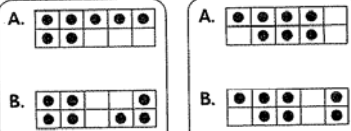
Taking numbers apart with ease (fluency) is critical to the effectiveness.



MAKING TENS

$$\begin{array}{l} 6 + 7 = (3 + 3) + 7 \\ 3 + (3 + 7) \\ 3 + 10 = 13 \end{array}$$

Break numbers apart to quickly make ten



$$\begin{array}{l} 5 + 3 + 5 + 4 + 7 \\ 9 + 5 + 8 + 2 + 1 \\ 4 + 5 + 6 + 3 + 7 \end{array}$$

Develop number fluency with combinations that make ten.

How many more are needed to make a group of ten?

$$26 + 27$$

More Addition Strategies

Breaking Numbers into Place Value
Break addends into expanded form and combine like place values.

$$\begin{array}{rcl} (20 + 6) + (20 + 7) \\ 20 & + & 20 = 40 \\ 6 & + & 7 = 13 \\ 40 + 13 = 53 \end{array}$$

WHAT is a Rekenrek?

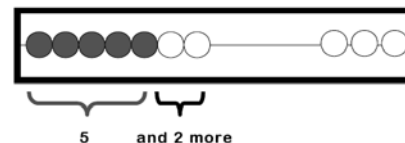
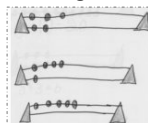
Rekenrek is a name for a small abacus-like math learning tool. The name came from a man who lived in the Netherlands. The name comes from two Dutch words:

Boekenrek – which is a book rack or shelf
Rekenboek – which is a math book

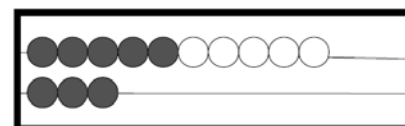
“Reken” (pronounced as raken) means doing math or arithmetic. In English the equivalent verb would be “to reckon.”

The meaning of reckon is *to think*, including *to consider, imagine, suppose, feel, deem, guess, or surmise*.

Reckon can also mean to *calculate, add up, total, count or number*.



Seven is seen as “5 and 2 more”

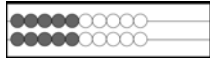


One row of 10
3 more

Thirteen is seen as “10 and 3 more”

WHAT is a Rekenrek?

Math Tool



- Two rows of stringed beads
- Repeating pattern of five beads of one color and five beads of another color on each row
- Begin with first five colored beads to help students *subitize*
- Use one row to build fluency to ten
- Use two rows to build fluency to twenty

Subitize? Fluency?

Subitize? Fluency?

- Subitize
 - To see the quantity of
- Fluency
 - Knowing how a number is composed and decomposed
 - Ability to use knowledge about the number
 - Provides flexibility and efficiency to solve problems

WHY Rekenreks?

Why Use Rekenreks?

- Reason about numbers
- Subitize
- Build fluency
- Compute using number relationship
- Assist in conceptual understanding



Rekenrek Video Clip

Number Talks Clip

- Number Talks: Helping Children Build Mental Math and Computation Strategies
 - By Sherry Parrish
 - Math Solutions, 2010



Sample Problems - K

Rekenreks with Number 4

- | | |
|----------------------------|----------------------------|
| A. 3 on top
1 on bottom | A. 1 on top
3 on bottom |
| B. 1 on top
3 on bottom | B. 2 on top
2 on bottom |
| C. 2 on top
2 on bottom | C. 4 on top
0 on bottom |

Sample Problem - K

Rekenreks with Number 9

- | | |
|----------------------------|----------------------------|
| A. 9 on top
0 on bottom | A. 4 on top
5 on bottom |
| B. 8 on top
1 on bottom | B. 3 on top
6 on bottom |
| C. 7 on top
2 on bottom | C. 2 on top
7 on bottom |
| D. 6 on top
3 on bottom | D. 1 on top
8 on bottom |

Sample Problems – Grade 1

Counting All/Counting On

- | | |
|----------------------------|----------------------------|
| A. 9 on top
3 on bottom | A. 8 on top
2 on bottom |
| B. 9 on top
4 on bottom | B. 8 on top
4 on bottom |
| C. 9 on top
5 on bottom | C. 8 on top
6 on bottom |

Sample Problems – Grade 1

Doubles/Near Doubles

- | | |
|----------------------------|----------------------------|
| A. 5 on top
5 on bottom | A. 6 on top
6 on bottom |
| B. 7 on top
5 on bottom | B. 8 on top
6 on bottom |
| C. 5 on top
5 on bottom | C. 6 on top
6 on bottom |
| D. 5 on top
3 on bottom | D. 6 on top
4 on bottom |

Sample Problems – Grade 1

Making Tens

- | | |
|----------------------------|----------------------------|
| A. 9 on top
1 on bottom | A. 7 on top
3 on bottom |
| B. 9 on top
3 on bottom | B. 7 on top
4 on bottom |
| C. 9 on top
5 on bottom | C. 7 on top
5 on bottom |

Where to begin?

GETTING STARTED

- ☑ **Limit the time** to 5 - 15 minutes
- ☑ Start with **smaller** problems to solicit multiple strategies
- ☑ **Suggest a strategy** from a previous student if a student is having difficulty getting started
- ☑ **Be patient** as you implement Math Talks



Math Talks



MATH TALK ESSENTIALS

1. Provide appropriate **wait time** for the majority of students to access the problem
2. Consider **all answers**; incorrect answers are OK
 - a. **How** did the student solve the problem?
 - b. **Does it fit** with the solution strategy?
3. Encourage **student communication**
4. Create a room environment that allows for **informal observations** and **interactions**
5. **Anticipate** potential strategies and student responses

Math Talks

KEY COMPONENTS

- Classroom Environment
- Classroom Discussions
- Role of the Teacher
- Role of Mental Math
- Purposeful Computation Problems



Math Talks

Math Talks

What Makes it a *Math Talk* as opposed to a lesson?

- **Understanding** how numbers work, rather than learning various skills
- Empowers students to examine problems in their **own way**
- **Short term practice** toward long term goals
- Increase difficulty levels - encourage students to find **more efficient ways** to solve problems
- Never expect students to see the problem the teacher's way
- **Not predictable**
- Does not replace current curriculum or lesson; only **10-15 minutes of each day**



21ST CENTURY

MATHEMATICS SKILLS NEEDED

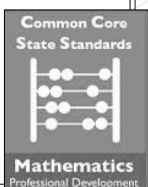
- Ability to reason about quantitative information
- Possess number sense
- Check for reasonableness of answers
- Discern whether numbers make sense
- Determine whether numbers are applicable to specific situations
- Communicate solutions to problems
- Compute accurately and efficiently



COMMON CORE

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Standards of Mathematical Practice

BENEFITS OF MATH TALK

- Investigate and apply mathematical relationships (SMP 2,3,7,8)
- Consider and test other strategies to see if they make sense (SMP 1)
- Clarify thinking (SMP 1,2)
- Build a repertoire of efficient strategies (SMP 1,3,5,8)
- Make decisions about choosing efficient strategies for specific problems (SMP 5,7,8)



Selecting Problems

1. Problems students are having trouble with.
2. Give students rich experiences in solving a variety of problems
3. Simple problems that allow students to move to more mature strategies
4. Interesting, engaging, and understandable for students



Results

Seeing Change

- Students can solve **more difficult** problems than they anticipated.

"I was really surprised what kids can do without any help from me at all... And even now when some of the kids tell you how they solve a problem, you have to admire that, because in your wildest dreams you wouldn't have thought about doing it that way."

Jennifer Beard, first-grade teacher
Children's Mathematics, CGI (90)

DOOR PRIZE!!! ☺



I love
I know I can use.....
I'd like to try...



I wonder
I'd like to know more about ...
Could this help? ...

Resources

- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. *Children's Mathematics, Cognitively Guided Instruction*. Heinemann, 1999.
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